The cohomology of the punctured tubular neighborhood of the “fiber at infinity” of an arithmetic variety can be computed by means of a complex first introduced by C. Consani in her Ph.D thesis. At archimedean infinity (and for arithmetic purposes i.e. for the description of the archimedean local factor in the Hasse-Weil L-function), this complex replaces Steenbrink's complex whose hypercohomology computes the cohomology of the universal fibre of a (semi-stable) degeneration over a disc. The nearby cycles complex associated to this degeneration carries a monodromy operator N and we can show that the graded pieces of the filtration on the cohomology of the nearby cycles complex by $Ker(N^j)$, $j \geq 0$, are isomorphic to the cyclic homology of a sheaf of differential operators (using some results of M. Wodzicki). Further, we can show that, under this isomorphism, Connes' periodicity operator in cyclic homology coincides with the (logarithm of) the monodromy on the nearby cycles complex.
In this talk, we will do the same at archimedean infinity, where we have to work with “global sections” rather than with sheaves ; and therefore show that there is a natural map from the cyclic cohomology of the ring of differential operators to the graded pieces of a filtration on the cohomology of the fibre at infinity, and that ; in this framework, the periodicity operator in cyclic cohomology is again the counterpart of the monodromy operator on Consani's complex. The switch between cyclic homology and cohomology is a consequence of the fact that the monodromy operators on the nearby cycles complex and on Steenbrink's complex are equal only up-to homotopy. This is followed up by defining a complex with monodromy that plays the role of a nearby cycles complex for the fibre at infinity. Again, the monodromy operator on the latter is equivalent to the monodromy on Consani's complex up-to homotopy.