

In the 1930's Leonard Carlitz described the first Drinfeld module associated to the polynomial ring  $\mathbb{F}_q[T]$  with coefficients in a finite field with  $q = p^t$  elements ( $p$  rational prime). Carlitz was able to express in an elementary fashion the associated exponential function (nowadays called "the Carlitz exponential") which is an entire  $\mathbb{F}_q$ -linear function. An interesting property of this exponential is that its Taylor expansion has only non-zero terms of the form  $z^{q^i}$  for  $i = 0, 1, \dots$ . By using the logarithmic derivative of this function, Carlitz was able to prove an analog of Euler's famous result on the values of the Riemann zeta-function at even integers. Then, using the denominators of the above exponential, Carlitz created a "factorial" element  $\Pi(j)$  based on the  $q$ -adic expansion of  $j$ . He then introduced what we call "Bernoulli-Carlitz" elements  $BC_i$  (i.e., certain rational functions in  $\mathbb{F}_q[T]$ ) and proved a very mysterious "von Staudt" result of the following form : a prime  $f$  divides the denominator of  $BC_i$  if and only if

1.  $q^{\deg f} - 1$  divides  $i$ .
2. the sum of the  $p$ -adic digits of  $i$  must equal  $\deg(f)t(p - 1)$ .  
(where  $t$  is defined above by  $q = p^t$ ).

While the first condition is very classical, the second is highly mysterious. In the "characteristic  $p$  world" of Drinfeld modules, one works with arbitrary  $A$  (arising from any global field over  $\mathbb{F}_q$  and any closed point  $\infty$ ) of which  $\mathbb{F}_q[T]$  is the simplest example. Carlitz's results were the first in a long chain of results making  $A/\mathbb{F}_q$  look very much like the integers numbers (for what concerns the existence of  $L$ -series, gamma functions, modular forms, periods etc.). In this, the work is adjoint to the notion of making the integers look very much like  $A/\mathbb{F}_q$  via the "field"  $\mathbb{F}_1$  which lies "below"  $\mathbb{Z}$ .... The polynomial ring  $\mathbb{F}_q[T]$  is Euclidean like the integers but is not a good representative of general  $A$  with non-trivial class number. For instance, it appears that the "trivial zeroes" of zeta functions have a very interesting non-classical behavior for general  $A$ . Calculations due to D.Thakur and J.Diaz-Vargas show that this behavior occurs for  $-i$  where  $i$  has bounded sum of  $p$ -adic digits. So one is drawn to Carlitz's second condition above. Experiments done with LHC (= Long Hard Calculations!) may give us clues as to what to expect for a general von Staudt result as well as deeper insight into the behavior of these char  $p$  valued functions.