In the 1930’s Leonard Carlitz described the first Drinfeld module associated to the polynomial ring $\mathbb{F}_q[T]$ with coefficients in a finite field with $q = p^t$ elements ($p$ rational prime). Carlitz was able to express in an elementary fashion the associated exponential function (nowadays called “the Carlitz exponential”) which is an entire $\mathbb{F}_q$-linear function. An interesting property of this exponential is that its Taylor expansion has only non-zero terms of the form $z^{q^i}$ for $i = 0, 1, \ldots$. By using the logarithmic derivative of this function, Carlitz was able to prove an analog of Euler’s famous result on the values of the Riemann zeta-function at even integers. Then, using the denominators of the above exponential, Carlitz created a “factorial” element $\Pi(j)$ based on the $q$-adic expansion of $j$. He then introduced what we call “Bernoulli-Carlitz” elements $BC_i$ (i.e., certain rational functions in $\mathbb{F}_q[T]$) and proved a very mysterious “von Staudt” result of the following form: a prime $f$ divides the denominator of $BC_i$ if and only if

1. $q^{\deg f - 1} \mid i$.
2. The sum of the $p$-adic digits of $i$ must equal $\deg(f)(p - 1)$.

(Where $t$ is defined above by $q = p^t$).

While the first condition is very classical, the second is highly mysterious. In the “characteristic $p$ world” of Drinfeld modules, one works with arbitrary $A$ (arising from any global field over $\mathbb{F}_q$ and any closed point $\infty$) of which $\mathbb{F}_q[T]$ is the simplest example. Carlitz’s results were the first in a long chain of results making $A/\mathbb{F}_q$ look very much like the integers numbers (for what concerns the existence of $L$-series, gamma functions, modular forms, periods etc.). In this, the work is adjoint to the notion of making the integers look very much like $A/\mathbb{F}_q$ via the “field” $F_1$ which lies “below” $\mathbb{Z}$. The polynomial ring $\mathbb{F}_q[T]$ is Euclidean like the integers but is not a good representative of general $A$ with non-trivial class number. For instance, it appears that the “trivial zeroes” of zeta functions have a very interesting non-classical behavior for general $A$. Calculations due to D.Thakur and J.Diaz-Vargas show that this behavior occurs for $-i$ where $i$ has bounded sum of $p$-adic digits. So one is drawn to Carlitz’s second condition above. Experiments done with LHC (= Long Hard Calculations!) may give us clues as to what to expect for a general von Staudt result as well as deeper insight into the behavior of these char $p$ valued functions.