

In this talk I will start with a short but useful demonstration. Having an algebra admitting a compatible action of a Hopf algebra and an invariant trace allows us to explicitly derive all of the formulae for the structure maps of the cocyclic module of the Hopf algebra. Having such an action also yields a characteristic map from Hopf-cyclic cohomology of the Hopf algebra into the ordinary cyclic cohomology of the underlying Algebra. Connes and Moscovici in their ground breaking paper [Comm. Math. Phys. 198 (1998), no. 1, 199–246] where they defined the Hopf-cyclic cohomology, obtained the geometric characteristic classes of codimension-1 foliations (namely the transverse fundamental class and the Godbillon-Vey class) from the Hopf-cyclic cohomology of the universal Hopf algebra of such foliations using this characteristic map. I will also present a short history of how this particular characteristic map evolved into a pairing between Hopf-cyclic cohomology of the Hopf algebra and the underlying algebra. One can compute cyclic cohomology of cyclic and cocyclic modules with a wide variety of methods each of which is suitable for a different purpose. This follows from the fact that there are very different homotopical localizations of the category of cyclic and cocyclic modules. I will show how these different localizations resulted in seemingly very different pairings of cyclic cohomology of Hopf algebras and ordinary algebras. Once cyclic cohomologies of different kinds (including the ordinary cyclic and Hopf-cyclic cohomologies) are placed in their proper homotopical context, I will construct a universal pairing in the derived category of (co)cyclic modules. I will also show that in the presence of a Hopf algebra acting on an algebra, there are comparison natural transformations from the universal pairing to other pairings constructed in the literature. This allows us to decide which of these pairings, and under which conditions, are isomorphic using a differential calculus of derived functors.