

A few years ago Manin proposed the use of noncommutative tori as a possible geometric framework under which to approach the explicit class field theory problem for real quadratic fields. Noncommutative tori are defined in terms of their algebras of functions, these algebras are noncommutative analogs of algebras of functions on the classical torus. Morphisms between noncommutative tori are given by Morita equivalences between the corresponding algebras. By considering the rank map induced by the canonical trace in a noncommutative torus these Morita type morphisms can be analyzed in terms of their images as morphisms between the K-theory groups of the corresponding algebras. A noncommutative torus has nontrivial Morita type endomorphisms if and only if the image under the rank map of the ring of such endomorphisms is an order in a real quadratic field. In this case the corresponding torus is called a real multiplication noncommutative torus. The strong analogy with the theory of elliptic curves suggest that noncommutative tori may play a role in the study of real quadratic fields analogous to the role played by elliptic curves in the theory of imaginary quadratic fields. Noncommutative geometry has been used to approach class field theory within the context of quantum statistical mechanics, in particular quantum statistical mechanical system whose symmetries are given by Galois groups have been constructed. Using this approach it is possible to recover the explicit class field theory of the field of rational numbers as well as the explicit class field theory of quadratic imaginary fields. Algebraic numbers generating the maximal abelian extension of the base field are obtained as values of extremal equilibrium states on elements of an arithmetically defined subalgebra of the algebra of observables of the system. The action of the Galois group on equilibrium states is intertwined with its action on these algebraic values. The recently developed theory of endomotives provides a unified framework appropriate to approach the class field theory of number fields from the perspective of quantum statistical mechanics. In the zero dimensional case endomotives correspond to inductive limits of finite dimensional algebras twisted by the action of an abelian semigroup of endomorphisms. An important class of examples of zero-dimensional endomotives is obtained from the action of self maps of algebraic varieties. The arithmetic structures of the quantum statistical mechanical systems encoding the explicit class field theory of the rational numbers and quadratic imaginary fields arise as examples of endomotives defined in this way. Extending this class of examples in order to consider the action of self maps of abelian varieties on spaces of sections of ample line bundles over them may provide valuable information of arithmetic nature. Whether there are endomotives arising in a natural way from noncommutative tori with real multiplication becomes then an important question. A natural candidate for the semigroup of endomorphisms defining such endomotives is furnished by Morita type morphisms of the corresponding noncommutative torus. The abelian part of these algebraic endomotives should come from the period pseudo lattice of the real multiplication noncommutative torus viewed as an order in the corresponding real quadratic field. The action of the Galois group should commute then with the action of algebraic endomorphisms furnished by integers in this order.